Multiple Mutation Strategies Differential Evolution With the Best Individuals Allocated to the Best Performer Among the Strategies

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Abstract

Real parameter single objective optimization has been a prominent field for these decades. Recently, long-term search of real parameter single objective optimization is widely concerned based on the fact that solving difficulty always scales exponentially with the increase of dimensionality of solution space. So far, a number of population-based metaheuristics have been proposed. Among the algorithms, IMODE - a differential evolution algorithm based on three mutation strategies and the binomial or exponential crossover - demonstrates good performance. In this paper, based on IMODE, we propose multiple mutation strategies Differential Evolution with the Best Individuals allocated to the Best performer among the Strategies - BIBSDE - by revising IMODE. Altogether, we make five revisions in algorithm behavior and a change in parameter setting. The most important revision is that, during execution, for the next generation, the current best individuals are allocated to the best performer among the three mutation strategies as reward. Experimental results show that our BIBSDE performs better or at least not worse than existing population-based metaheuristics for long-term search. Besides, each measure proposed by us is effective for enhancement.

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1. Introduction

Real parameter single objective optimization involves minimizing or maximizing an objective function by finding the optimal decision vector within the solution space. Over the years, a variety of population-based metaheuristics have been proposed for the purpose. Among the types, Differential Evolution (DE) has shown remarkable performance. For example, in the very influential competitions on real parameter single objective optimization held in the Congress of Evolutionary Computation (CEC) series, most winners are based on DE.

DE is with the three main operators - mutation, crossover, and selection. Initially, individuals in the population of DE, target vectors $\vec{x}_{i,0} = (x_{1,i,0}, x_{2,i,0}, ..., x_{D,i,0})$, where $i \in \{1, 2, ..., NP\}$, NP denotes population size, and D represents dimensionality, are produced. At most occasions, the initialization is randomly performed. After that, evolution is done for generations. In the gth generation, mutant vectors $\vec{v}_{i,g}$ are generated through mutation. So far, many mutation strategies have been proposed in literature. Here, DE/best/1, one of the early mutation strategies,

$$\vec{v}_{i,g} = \vec{x}_{best,g} + F \cdot (\vec{x}_{r1,g} - \vec{x}_{r2,g}), \tag{1}$$

are given in Equation 1 for example. In the equation, $\vec{x}_{best,g}$ denotes the best target vector in the gth generation of the population. $r1 \in \{1, 2, ..., NP\}$ and $r2 \in \{1, 2, ..., NP\}$ are random produced. Here, $r1 \neq r2 \neq i$. Besides, F is the parameter scaling factor. Based on early mutation strategies, more ones are proposed. For example, DE/current-to-ϕbest/1,

$$\vec{v}_{i,g} = \vec{x}_{i,g} + F_i \cdot (\vec{x}_{\phi,g} - \vec{x}_{i,g} + \vec{x}_{r1,g} - \vec{x}_{r2,g}), \tag{2}$$

and DE/current-to-ϕbest/1 with archive,

$$\vec{v}_{i,g} = \vec{x}_{i,g} + F_i \cdot (\vec{x}_{\phi,g} - \vec{x}_{i,g} + \vec{x}_{r1,g} - \vec{x}_{r3,g}), \tag{3}$$

are very complicated mutation strategies, while DE/weighted-rand-to-ϕbest/1 -

$$\vec{v}_{i,g} = F_i \cdot \vec{x}_{r1,g} + (\vec{x}_{\phi,g} - \vec{x}_{r2,g}), \tag{4}$$

is very a special one. In Equations 2 to 4, $r1 \neq r2 \neq r3 \neq i$. $r1 \in \{1, 2, ..., NP\}$, $r2 \in \{1, 2, ..., NP\}$. Meanwhile, $r3 \in \{1, 2, ..., NP, ..., NP+|A|\}$, where $|A|$ is the size of an external archive for collecting the target vectors eliminated from the population in selection. Thus, $\vec{x}_{r1,g}$ and $\vec{x}_{r2,g}$ are target vectors randomly chosen from the gth generation of the population, while $\vec{x}_{r3,g}$ are randomly

chosen from the generation or the external archive. In addition, $\vec{x}_{\phi,g}$ denotes a target vector among the $NP \cdot \phi$ best ones in the gth generation, where $\phi \in (0,1)$. It can be seen that, in the three equations, each mutant vector $\vec{v}_{i,g}$ has its own scaling factor F_i. After mutation, trial vectors $\vec{u}_{i,g} = (u_{1,i,g}, u_{2,i,g}, ..., u_{D,i,g})$ are generated based on $\vec{x}_{i,g}$ and $\vec{v}_{i,g}$ by crossover. The binomial crossover,

$$u_{j,i,g} = \begin{cases} v_{j,i,g}, & if \ rand(0,1) \leq CR \ or \ j = j_{rand}, \\ x_{j,i,g}, & otherwise, \end{cases} \tag{5}$$

where $CR \in [0,1]$ is the crossover rate, and j_{rand} is an integer randomly generated from the range $[1, D]$ to ensure that $\vec{u}_{i,g}$ has at least one component from $\vec{v}_{i,g}$, is used in most DE algorithms. However, other crossover strategies, e.g., the binomial or exponential crossover,

$$u_{j,i,g} = \begin{cases} \begin{cases} v_{j,i,g}, & if \ rand(0,1) \leq CR \ or \ j = randn(i), \\ x_{j,i,g}, & otherwise, \end{cases} & if \ rand(0.1) \leq p \\ \begin{cases} v_{j,i,g}, & for \ j = \langle l \rangle_D, \langle l+1 \rangle_D, ..., \langle l+L-1 \rangle_D \\ x_{j,i,g}, & for \ all \ other \ j \in [1, D], \end{cases} & otherwise, \end{cases} \tag{6}$$

do exist. In Equation 6, the parameters not explicitly mentioned above are explained below. p is the probability of the binomial manner. That is, the exponential manner has the probability $1 - p$. Here, $l \in \{1, 2, ..., D\}$ is randomly decided during execution, while $L \in \{1, 2, ..., D\}$, as a parameter, requires to be set before execution. In general, mutation and crossover are joined together under the term trial vector generation strategy. For selection,

$$\vec{x}_{i,g+1} = \begin{cases} \vec{u}_{i,g}, & if \ f(\vec{u}_{i,g}) \leq f(\vec{x}_{i,g}), \\ \vec{x}_{i,g}, & otherwise, \end{cases} \tag{7}$$

where $f(\vec{u}_{i,g})$ and $f(\vec{x}_{i,g})$, obtained by function evaluation, represent the fitness of $\vec{u}_{i,g}$ and $\vec{x}_{i,g}$, respectively.

To further enhance the performance of DE for real parameter single objective optimization, at least four directions,

- Innovation of trial vector generation strategy [1, 2, 3, 4, 5, 6, 7, 8, 9, 10];

- Adaption of control parameter setting [11, 2, 9];

- Hybridization with search techniques other than DE [12, 13, 14, 15]; and

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- Ensemble of multiple trial vector generation strategies or even DE algorithms [16, 11, 17, 18, 19, 20, 21],

are pursued by researchers.

So far, researches are centered on two types of search in the field of real parameter single objective optimization. Traditional search, which means the maximum number of function evaluations ($MaxFES$) as the termination criterion scales linearly with increase of D, has been studied for decades. In detail, $D \in \{10, 30, 50, 100\}$, while $MaxFES = D \cdot 1.00E + 04$. The CEC competitions before 2020 concentrate on traditional search. In traditional search, it is very easy to obtain the optimum of functions when $D = 10$. With the increase in D, it becomes more and more difficult to obtain that. After all, in practice, solving difficulty often scales exponentially with the increase of D. Based on the above fact, recently, long-term search, which means $MaxFES$ scales linearly with increase of D, begins to be concerned. The CEC 2020, 2021, and 2022 competitions concentrate on long-term search. Up to now, the minimum and the maximum of D are 10 and 20, respectively. When $D = 10$, $MaxFES = 1.00E + 06$. When D goes to 20, $MaxFES$ becomes $1.00E + 07$. Provided that $D > 20$, the large value of $MaxFES$ may make experiment become infeasible at the present stage. In long-term search, performance of algorithm demonstrates much less variation with the increase of D. Long-term search is reasonable and worth studying although $D > 20$ still can hardly be supported in experiment. With the development of hardware, larger value may be taken for D in the future.

Since 2020, population-based metaheuristics suitable for long-term search begin to be proposed. In the algorithms, measures are implemented to slow down convergence velocity. Therefore, searching becomes broader than before to resist stagnation. Among the algorithms, the Improved Multi-Operator DE algorithm (IMODE) [16] is based on the three mutation strategies demonstrated in Equations 2-4 and the binomial or exponential crossover shown in 6. The algorithm, an ensemble of multiple trial vector generation strategies, is the top performer in the CEC 2020 competition - the first competition of long-term search. In each generation, for every position in the population of IMODE, the difference in fitness between target vector and trial vector is computed. Then, $\overline{\Delta f}_k$, the average improvement from target vector to trial vector of all the positions controlled by the kth mutation strategy can be obtained. After that, the controlling ratio of the kth mutation strategy

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c_k is computed as below,

$$c_k = \frac{\overline{\Delta f_k}}{\sum_{i=1}^{3} \overline{\Delta f_i}}, \tag{8}$$

for the next generation. According to Equation 8, better performance leads to larger controlling ratio. Moreover, a measure is adopted to make $0.1 \leq c_k \leq 0.9$. In detail, provided that $c_m < 0.1$ ($m \in 1, 2, 3$), c_m is turned to 0.1 by reducing c_{max}. In IMODE, the adaptive setting of the scaling factor F and the crossover rate CR, as well as the maintenance of the external archive, is according to [22]. In detail, F_i - F of the ith position in the population is independently produced based on a Cauchy distribution with location μF and scale 0.1,

$$F_i = randc_i(\mu F, 0.1). \tag{9}$$

Here, if $F_i > 1$, the parameter is truncated to 1. If $F_i < 0$, the parameter is regenerated. μF in Equation 9 is obtained as

$$\mu F = (1 - c) \cdot \mu F + c \cdot mean_L(S_F), \tag{10}$$

where c is a constant between 0 and 1, $mean_L$ denotes Lehmer mean, and S_F is the set of all successful values of F in the previous generation. Meanwhile, CR_i - CR of the ith position is independently generated based on a normal distribution of mean μCR and standard deviation 0.1,

$$CR_i = randn_i(\mu CR, 0.1). \tag{11}$$

Also, CR_i is truncated to $[0, 1]$. μCR in Equation 11 is obtained as

$$\mu CR = (1 - c) \cdot \mu CR + c \cdot mean_A(S_{CR}), \tag{12}$$

where $mean_A$ denotes arithmetic mean, and S_{CR} is the set of all successful values of CR in the previous generation. For selection, Equation 7 is used. In the next generation, $c_k \cdot NP$ individuals are allocated randomly to the kth mutation strategy. In the final stage of evolution, a local search method based on sequential quadratic programming is executed beside the three main operators.

It can seen that, in each generation of IMODE, individuals may be allocated to any one of the mutation strategies without any restriction. That is, the allocation is completely random. Therefore, no position in the population may be persistently controlled by a mutation strategy. By this means, convergence is slowed down. Thus, the probability of algorithm falling into stagnation is reduced. Hence, IMODE is fit for long-term search. Nevertheless, The simple scheme with excessive randomness for allocating individuals may not be the best choice.

To obtain better solution, IMODE needs to be further improved. For example, the allocation of individuals to the three mutation strategies deserves to be further studied. Our direction of study is to reduce the randomness in the allocation to some extent. For the purpose, it is feasible to consider regarding the best individuals as reward to the best performer among the mutation strategies. Of course, other aspects of algorithm may also be adjusted. The above is our motivation.

In this paper, based on IMODE, we propose a new DE ensemble, multiple mutation strategies Differential Evolution with the Best Individuals allocated to the Best performer among the Strategies - BIBSDE. In our ensemble, the same mutation strategies and crossover strategy as IMODE are still used. Besides, the criterion for evaluating performance of mutation strategy in IMODE is still employed. Meanwhile, the original method for deciding the controlling ratio of mutation strategy defined in the IMODE is followed. However, we revise IMODE as below. Firstly, the Linear Population Size Reduction (LPSR) is replaced by Non-Linear Population Size Reduction (NLPSR) [23]. Then, for NLPSR, we propose that the reduced individuals are not the worst ones, but the ones just better than the worst ones. Thirdly, for the mutation strategies, the ratio of best individuals - ϕ - in Equations 2-4 descends linearly during the whole course. Fourthly, in the second half of execution, CR is uniformly set for every positions and descends linearly. More importantly, for the next generation, the current best individuals are allocated to the mutation strategy with the highest controlling ratio. That is, the current best individuals are given to the best performer among the three mutation strategies as reward.

To evaluate performance of the proposed algorithm for long-term search, experiment is executed based on the CEC 2020 and 2022 benchmark test suites [24, 25], and problems selected from the CEC 2011 suite of real-world optimization problems [26]. The two CEC benchmark test suites are designed for long-term search, while earlier ones, such as the CEC 2017 benchmark test suite [27], are for traditional search. For the reason given above, just the problems lower in dimensionality among the CEC 2011 suite of real-world optimization ones are involved in experiment. In the first experiment, our algorithm is compared with seven population-based metaheuristics suitable for long-term search based on the CEC 2020 and 2022 benchmark test suites when $D = 10$ and $D = 20$. Details of these peers are provided in related work. Furthermore, in the second experiment, BIBSDE and four algorithms further chosen from the peers are compared based on the selected CEC 2011 real-world optimization problems. Results of the above two experiments show that our algorithm is competitive. Then, effectiveness of each measure proposed for BIBSDE is verified

based on the CEC 2020 benchmark testing suite when $D = 15$.

The structure of the rest of the paper is outlined as follows. Related work is summarized in Section 2. Then, the proposed algorithm is given in Section 3. In Section 4, experiment is carried out. Finally, a conclusion is dealt with in Section 5.

2. Related work

Existing algorithms for long-term search of real parameter single objective optimization is reviewed in this section. Although quantity of algorithms for long-term search is still less than that of algorithms for traditional search, the former quantity is increasing rapidly.

The top three algorithms in the CEC 2020 competition are IMODE, AGSK [23], and j2020 [1]. As mentioned above, in IMODE, the three mutation strategies seize individual during evolution. Besides, IMODE employs the binomial or exponential crossover. AGSK is an adaptive version of a new type of population-based metaheurisic - gaining-sharing knowledge algorithm. In AGSK, junior gaining-sharing knowledge phase and senior gaining-sharing knowledge one are executed on different dimensions, respectively. The controlling rate of each phase is decided by an adaptive scheme. Among the early DE algorithms with self-adapting mechanism for control parameter, jDE [28] is very famous. The type of DE ensemble with two subpopulations, jDE100 [29], is proposed in based on jDE, As a variant of jDE100, j2020 is with a crowding mechanism and a scheme to choose individual for mutation from both subpopulations.

In the CEC 2021 suite, each function is parameterized with bias, rotation, and translation. Thus, in the CEC 2021 competition, algorithms are ranked for different types of cases - non-shifted ones, shifted ones, non-rotated shifted ones, and rotated shifted ones, respectively. APGSK-IMODE [14] ranks first for non-shifted cases, while jDE-21 [3] ranks first for non-rotated shifted ones. Moreover, NL-SHADE-RSP [5] ranks first for both shift cases and rotated shifted ones. APGSK-IMODE is the ensemble of AGSK and IMODE. In the ensemble, each component algorithm is responsible for controlling a subpopulation within the overall population. Additionally, the probability of each component algorithm running is determined based on the solution quality. At intervals, the two subpopulation share the current best individual. Based on j2020, jDE-21 employs a restart mechanism to maintain diversity. Meanwhile, jDE-21 adopts the crowding mechanism and the scheme to choose individual for mutation in j2020 after revision. NL-SHADE-RSP is a revised

version of L-SHADE-RSP [30] - a good performer in the CEC 2018 competition. In NL-SHADE-RSP, the selective pressure is fine-tuned by making slight adjustments to the settings of the mutation strategy. Furthermore, NL-SHADE-RSP incorporates an automatic tuning mechanism for the archive usage probability.

The top three algorithms in the CEC 2022 competition are EA4eig [15], NL-SHADE-LBC [9], and NL-SHADE-RSP-MID [8]. EA4eig is the ensemble of four population-based metaheruistics - CoBiDE [31], IDEbd [32], CMA-ES [33], and jSO [34]. In the ensemble, the Eigen approach proposed in CoBiDE is extended to all the constituent algorihtms. NL-SHADE-LBC integrates selective pressure, parameter adaptation with linear bias change, the current-to-pbest mutation strategy, resampling for bound constraint, as well as NLPSR. NL-SHADE-RSP-MID is built upon NL-SHADE-RSP by introducing measures - updating the best-so-far result by calculating the midpoint of the population at each generation, a triggering scheme for restart, and dividing the population by the k-means clustering.

In [21], APGSK-IMODE-FL is proposed by revising APGSK-IMODE. In APGSK-IMODE-FL, the two subpopulations are still controlled by APGSK and IMODE, respectively. However, a more complicated scheme based on both lifetime and fitness is used to choose individuals for exchange. In [20], AMCDE is proposed based on IMODE. In AMCDE, there exist two states, monopoly and competition. In the former state, just one of the three mutation strategy controls all individuals. In the latter state, the three mutation strategies compete with each others to become the next monopolizer.

Population reduction has been widely applied for improving solution in all of the above algorithms. LPSR, the method of population reduction, having been widely used for years, is employed in IMODE and j2020, while NLPSR is used in the rest of the algorithms. According to the latter scheme given in Equation 13,

$$NP = round((\frac{FES}{MaxFES})^{1-\frac{FES}{MaxFES}} \cdot (NP_{min} - NP_{max}) + NP_{max}), \tag{13}$$

where FES denotes the consumed number of function evaluations, NP_{max} and NP_{min} represent the initial value and the final one of NP, respectively, the population size decreases rapidly in the early stage but slowly in the late stage. According to our analysis based on pre-experiment, at least for long-term search, NLPSR may lead to better solution than LPSR.

3. Methodology

Although IMODE has good performance, better solution may be obtained after revision. Firstly, we focus on population size reduction. In exist algorithms, the individuals reduced by either NLPSR or LPSR are always the worst ones. Then, we investigate parameter setting of trial vector generation strategy. For both mutation and crossover, parameter setting may be revised for improvement. Most importantly, we further study the allocation of individual to mutation strategy. The scheme for the allocation in IMODE is with excessive randomness and needs to be revised. In brief, we plan to improve IMODE from the above three aspects.

At the first aspect, now that most of the population-based metaheuristics for long-term search are with NLPSR, LPSR is replaced by NLPSR for enhancement. Moreover, for NLPSR, although Equation 13 is still used, instead of reducing the worst individuals in the population, the individuals that are just better than the worst ones are targeted. Let NP_{before} and NP_{after} be the value of NP before and after a time of reduction, respectively. In reduction, $NP_{before} - NP_{after}$ individuals just better than the worst $NP_{before} - NP_{after}$ ones are reduced. Thus, the worst individuals are still remained after NLPSR. However, at the final stage of evolution, the population become small in size. In this case, the best individual in the population may be among the $NP_{before} - NP_{after}$ individuals just better than the worst $NP_{before} - NP_{after}$ ones. If the phenomenon is observed, the worst individuals are reduced.

The reason for reducing such individuals is given below. No matter which trial vector generation strategy is adopted, the selection method described in Equation 7 tends to drive individuals closer and closer to the best individual. However, this convergence towards the best individual leads to a decrease in population diversity. Indeed, it is often the case that the worst individuals in the population are the ones that differ the most from the best individual. By not reducing the worst individuals but targeting individuals just better than them for removal, it is likely that loss in diversity becomes less. Meanwhile, negative effect is insignificant. After all, individuals with good fitness are still preserved after reduction.

At the second aspect, adaptive setting for parameters is further emphasized. Firstly, the ratio of best individuals ϕ for the mutation strategies shown in Equations 2-4 is set adaptively. In detail,

$$\phi = (\phi_{min} - \phi_{max}) \cdot \frac{FES}{MaxFES} + \phi_{max}. \tag{14}$$

Moreover, adaptive setting of CR is revised. According to [5], the binomial crossover rate, CR_b, of

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NL-SHADE-RSP is set differently in the first half of execution and the second one. In detail,

$$CR_b = \begin{cases} 0, & if \ FES <= MaxFES \cdot 0.5, \\ 2 \cdot \frac{FES-0.5}{MaxFES}, & otherwise. \end{cases} \qquad (15)$$

Enlightened by Equation 15, CR setting in our algorithm is also differently set in the two stages. The original method of setting CR_i shown in Equation 11 and 12 is still used in the first half of execution, while a new method

$$CR = 2 \cdot (1 - \frac{FES}{MaxFES}), \qquad (16)$$

is for the second one. According to Equation 16, in the second half of execution, the crossover rate CR is set uniformly in all positions and descends linearly.

At the third aspect, a reward scheme is built for the allocation of individual to mutation strategy. Although the controlling ratio of each mutation strategy for the next generation is still computed based on the original method in IMODE, individuals are allocated as below. The best individuals are all given to the best performer among the mutation strategies. Meanwhile, the other individuals are randomly allocated to the other mutation strategies. Based on our reward scheme, the randomness in the allocation of individual to mutation strategy is reduced to some extent.

Details of our BIBSDE are given in Algorithm 1. BIBSDE has two more parameters, ϕ_{max} and ϕ_{min}, than IMODE. The changes in flow in our BIBSDE based on IMODE are listed below. As shown in Steps 7-11, CR setting is revised. According to Step 12, ϕ decreases linearly. In Step 20, the mutation strategy performing best is found. Then, in Step 21, the mutation strategy is given to the best individuals.

4. Experimental study

In the first experiment, our BIBSDE is compared with the seven algorithms introduced in Section 2 - IMODE, AGSK, APGSK-IMODE, NL-SHADE-RSP, MLS-L-SHADE, EA4eig, and AMCDE - based on the CEC 2020 and 2022 benchmark testing suites. Tables 1 and 2 are used to introduce the two suites briefly. Then, from the CEC 2011 real-world problems, we select the problems whose $D \leq 20$ to further compare BIBSDE with algorithms selected from the above peers. Table 3 shows all the problems selected by us. Meanwhile, $MaxFES$ is adjusted for long-term search. Table 4 gives $MaxFES$ for the selected CEC 2011 real-world problems, which scales exponentially with the

Algorithm 1 The pseudo-code of BIBSDE

Input: NP_{max}, NP_{min}, $MaxFES$, ϕ_{max}, and ϕ_{min}

Parameter: NP, FES, CR_i $(i = \{1, 2, ..., NP\})$, ϕ, c_k $(k = \{1, 2, 3\})$, and b

1: Initialize the initial generation of the population P_0

2: Evaluate the NP_{max} individuals in P_0

3: $NP = NP_{max}$, $FES = NP$

4: Allocate one third random individuals to the k-th mutation strategy among the ones given in Equations 2-4

5: **while** $FES <= MaxFES$ **do**

6:    F_i in Equations 2-4 is set according to Equations 9 and 10

7:    **if** $FES <= MaxFES \cdot 0.5$ **then**

8:       For Equation 6, CR_i is set according to Equations 11 and 12

9:    **else**

10:       For Equation 6, CR is set according to Equation 16

11:    **end if**

12:    ϕ is set according to Equation 14 for Equations 2-4

13:    For each individual, execute mutation according to the allocation

14:    Execute crossover based on Equation 6 to obtain trial vectors

15:    Evaluate the trial vectors

16:    $FES = FES + NP$

17:    Execute selection based on Equation 7 to obtain individuals of the next generation

18:    Calculate c_k based on Equation 8

19:    Make $0.1 \leq c_k \leq 0.9$

20:    Find b from $\{1, 2, 3\}$ to make $c_b = max(c_1, c_2, c_3)$

21:    Allocate the best $c_b \cdot NP$ individuals to the bth mutation strategy

22:    Randomly allocate the rest individuals so that $c_k \cdot NP$ individuals are allocated to the kth mutation strategy

23:    **if** $FES \geq 0.85 \cdot MaxFES$ **then**

24:       Apply the sequential quadratic programming to the best individual

25:       Update FES

26:    **end if**

27:    Execute the revised version of NLPSR based on Equation 13 to decrease NP

28: **end while**

29: Report solution

Table 1: Classification of the functions in the two CEC suites

Type	CEC 2020	CEC 2022
Unimodal functions	F1	F1
Basic functions	F2-F4	F2-F5
Hybrid functions	F5-F7	F6-F8
Composition functions	F8-F10	F9-F12

Table 2: Setting of the CEC 2020 and CEC 2022 benchmark test suites

D	MaxFES	
	2020	2022
5	5.00E+04	-
10	1.00E+06	1.00E+06
15	3.00E+06	-
20	1.00E+07	1.00E+07

Table 3: The problems selected from the CEC 2011 real-world ones

Problem number	Problem name	Dimensionality
P1	Parameter estimation for frequency modulated sound waves	6
P3	Bifunctional catalyst blend optimal control problem	1
P4	Optimal control of a non-linear stirred tank teacto	1
P8	Transmission network expansion planning problem	7
P10	Circular antenna array design problem	12
P11.3	Static economic load dispatch instance 1	6
P11.4	Static economic load dispatch instance 2	13
P11.5	Static economic load dispatch instance 3	15

increase of D, for the selected problems. In the third experiment, effectiveness of all the revisions is observed. In all cases, algorithm is executed 30 times for comparison.

Table 4: Value of $MaxFES$ for the problems selected from the CEC 2011 suite

D	$MaxFES$
$D \leq 5$	5.00E+04
$5 < D \leq 10$	1.00E+06
$10 < D \leq 15$	3.00E+06

4.1. Experimental Settings

The settings of all the algorithms involved in our experiment are listed in Table 5. In fact,

Table 5: Settings of the involved algorithms

Algorithm	Parameters
IMODE	$NP_{max} = D^2 \cdot 6$, $NP_{min} = 4$, $A_{rate} = 2.60$, $H = D \cdot 20$, $FES_{LS} = MaxFES \cdot 0.85$, and $p = 0.3$
AGSK	$NP_{max} = D \cdot 20$, $NP_{min} = 12$, $p = 0.05$, and $c = 0.05$
APGSK-IMODE	$NP_2^{max} = \frac{D \cdot 30}{4}$, $NP_2^{min} = 12$, $NP_1^{max} = D \cdot 30 - NP_2^{max}$, $NP_1^{min} = 4$, and $CS = 50$
NL-SHADE-RSP	$NP_{max} = 30D$, $M_{f,r} = 0.2$, $M_{CR,r} = 0.2$, and $n_A = 0.5$
MLS-L-SHADE	$NP_{max} = D \cdot 18$, $NP_{min} = 4$, $A_{rate} = 2.60$, and $H = D^2 \cdot 0.36$,
EA4eig	$NP_{max} = 100$ and $NP_{min} = 10$
AMCDE	$NP_{max} = D^2 \cdot 6$, $NP_{min} = 4$, $A_{rate} = 2.60$, $H = D \cdot 20$, $FES_{LS} = MaxFES \cdot 0.85$, and $p = 0.3$ $G_{n\_init} = 5$, $p_{bc1} = 0.4$, $p_{bc2} = 0.4$, $p_r = 0.01$, and $p_w = 0.2$
BIBSDE	Beside the parameter setting of IMODE, $p = 0.7$, $\phi_{max} = 0.4$, and $\phi_{min} = 0.2$

all the parameters used in IMODE still exist in BIBSDE. We use the original value of the most parameters but reset p for improvement. The new setting of p is considered in the following ablation experiment beside the three aspects of revision on algorithm. Besides, in BIBSDE, ϕ_{max} and ϕ_{min} are new parameters for one of our schemes.

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4.2. Comparison based on the CEC 2020 and 2022 benchmark test suites

In this experiment, BIBSDE is compared with the seven peers based on the CEC 2020 and 2022 benchmark test suites. The dimensionality D of functions is set to 10 and 20. In Tables 6-9, the results are listed with the Wilcoxon rank sum test at a 0.05 significance level. Furthermore, outcome of the Friedman test is given in Tables 10-13.

Table 6: The results for the CEC 2020 benchmark test functions when $D = 10$ with the Wilcoxon rank sum test. "+" or "−" denotes that the current result is significantly better or statistical worse than the result of BIBSDE in terms of the Wilcoxon rank sum test at a 0.05 significance level, respectively. Meanwhile, "≈" represents that there is no significant difference

Function	Average(standard deviation)							
	IMODE	AGSK	APGSK.IMODE	NL-SHADE-RSP	MLS-L-SHADE	EA4eig	AMCDE	BIBSDE
F1	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)
F2	3.93E+00 (3.41E+00)−	3.42E+01 (3.86E+01)−	7.67E+00 (8.40E+00)−	7.03E-01 (2.06E+00)≈	8.73E+00 (5.07E+00)−	4.42E+00 (5.10E+00)−	5.51E+00 (4.23E+00)−	7.84E-01 (1.28E+00)
F3	1.24E+01 (7.58E-01)−	9.20E+00 (4.20E+00)≈	1.19E+01 (6.59E-01)−	7.97E+00 (1.85E+01)≈	1.02E+01 (3.15E+00)−	1.07E+01 (1.22E+00)−	1.07E+01 (2.17E+00)−	1.00E+01 (2.34E+00)
F4	2.96E-03 (4.60E-03)≈	6.01E-01 (1.40E-01)−	1.17E-01 (8.30E-02)−	0.00E+00 (0.00E+00)+	0.00E+00 (0.00E+00)+	3.08E-01 (7.35E-02)−	0.00E+00 (0.00E+00)+	2.30E-03 (4.25E-03)
F5	4.23E-01 (3.88E-01)−	2.91E-01 (2.16E-01)−	4.49E-01 (4.70E-01)−	2.47E-01 (7.64E-02)≈	2.53E+00 (2.13E+00)−	5.55E-02 (9.36E-02)+	1.94E-01 (1.08E-01)≈	7.64E-02 (1.16E-01)
F6	1.20E-01 (7.22E-02)−	3.76E-01 (0.00E+00)−	8.33E-02 (1.05E-01)≈	6.90E-02 (5.83E-03)≈	3.94E-01 (2.13E+00)−	4.78E-02 (9.67E-02)+	4.81E-02 (5.15E-02)+	5.31E-02 (4.80E-02)
F7	6.76E-04 (7.30E-04)+	1.24E-03 (1.82E-03)−	7.42E-04 (8.16E-04)+	1.10E-03 (1.37E-06)−	1.36E-01 (1.50E+00)−	2.09E-02 (7.96E-02)−	7.84E-05 (1.28E-04)+	7.75E-04 (3.03E-03)
F8	2.72E+00 (7.46E+00)≈	2.39E+01 (3.03E+01)−	7.23E+01 (4.02E+01)−	3.29E+00 (5.70E+01)≈	1.28E+01 (1.95E+00)−	1.00E+02 (0.00E+00)−	1.04E+01 (1.74E+01)−	2.68E+00 (6.44E+00)
F9	4.15E+01 (4.38E+01)≈	7.00E+01 (4.66E+01)−	9.67E+01 (1.83E+01)−	6.44E+01 (2.28E+03)−	7.42E+01 (3.77E+00)−	2.56E+02 (9.65E+01)−	7.86E+01 (4.07E+01)−	4.00E+01 (4.98E+01)
F10	3.98E+02 (3.42E-12)−	3.18E+02 (1.34E+02)≈	3.78E+02 (7.55E+01)≈	3.68E+02 (8.25E+03)≈	3.68E+02 (8.92E+00)−	4.09E+02 (1.96E+01)−	3.68E+02 (9.08E+01)−	3.38E+02 (1.21E+02)
+	1	0	1	1	1	2	3	
−	5	7	6	2	8	7	5	
≈	4	3	3	7	1	1	2	

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Table 7: The results for the CEC 2020 benchmark test functions when $D = 20$ with the Wilcoxon rank sum test. "+" or "−" denotes that the current result is significantly better or statistical worse than the result of BIBSDE in terms of the Wilcoxon rank sum test at a 0.05 significance level, respectively. Meanwhile, "≈" represents that there is no significant difference

Function	Average(standard deviation)							
	IMODE	AGSK	APGSK_IMODE	NL-SHADE-RSP	MLS-L-SHADE	EA4eig	AMCDE	BIBSDE
F1	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)
F2	3.46E-01 (5.66E-01)−	1.41E+00 (1.10E+00)−	1.83E+00 (1.42E+00)−	1.98E-02 (4.36E-04)≈	1.27E+00 (9.83E+00)−	3.53E+00 (2.30E+00)−	1.25E+00 (1.33E+00)−	2.39E-02 (1.96E-02)
F3	2.05E+01 (1.24E-01)−	2.04E+01 (0.00E+00)≈	2.04E+01 (4.89E-02)−	2.04E+01 (0.00E+00)≈	2.07E+01 (1.37E+00)−	2.22E+01 (8.19E-01)−	2.04E+01 (0.00E+00)≈	2.04E+01 (0.00E+00)
F4	5.00E-01 (8.08E-02)−	7.89E-01 (9.00E-02)−	5.42E-01 (9.88E-02)−	0.00E+00 (0.00E+00)+	0.00E+00 (0.00E+00)+	7.24E-01 (1.32E-01)−	4.51E-01 (7.87E-02)−	4.35E-01 (7.40E-02)
F5	1.05E+01 (4.11E+00)≈	3.67E+01 (3.51E+01)−	9.57E+00 (6.87E+00)≈	1.37E+01 (9.28E+02)−	5.05E+00 (6.98E+00)+	1.26E+01 (2.97E+01)−	5.03E+00 (3.01E+00)+	1.15E+01 (4.52E+00)
F6	2.87E-01 (7.40E-02)−	4.49E+02 (2.89E-13)−	1.99E-01 (1.19E-01)≈	1.38E-01 (1.83E-03)+	3.39E-01 (7.83E+00)−	1.63E-01 (9.35E-02)≈	9.60E-02 (3.23E-02)+	1.96E-01 (5.99E-02)
F7	5.10E-01 (1.78E-01)−	4.29E-01 (1.81E-01)−	4.34E-01 (6.65E-01)≈	2.05E-01 (7.73E-03)+	6.28E-01 (1.51E+00)−	5.72E-01 (1.56E-01)≈	4.02E-01 (1.50E-01)−	2.59E-01 (9.27E-02)
F8	8.42E+01 (1.83E+01)≈	1.00E+02 (0.00E+00)−	1.00E+02 (2.31E-13)−	8.80E+01 (3.29E+02)≈	9.31E+01 (1.93E+00)−	1.00E+02 (2.12E-13)−	9.86E+01 (7.49E+00)−	8.21E+01 (2.21E+01)
F9	9.67E+01 (1.83E+01)≈	1.00E+02 (0.00E+00)≈	1.87E+02 (1.40E+02)−	1.00E+02 (3.43E-26)≈	1.00E+02 (1.06E+00)≈	4.06E+02 (3.24E+00)−	9.83E+01 (9.46E+00)≈	1.00E+02 (2.31E-13)
F10	4.00E+02 (4.44E-01)+	3.99E+02 (2.01E+00)+	4.06E+02 (5.78E+00)−	3.99E+02 (1.19E-01)≈	4.11E+02 (3.51E+00)−	4.06E+02 (1.53E-02)−	4.11E+02 (5.53E+00)−	4.01E+02 (5.02E+00)
+	1	1	0	3	2	0	2	
−	5	6	6	2	6	7	5	
≈	4	3	4	5	2	3	3	

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Table 8: The results for the CEC 2022 benchmark test functions when $D = 10$ with the Wilcoxon rank sum test. "+" or "−" denotes that the current result is significantly better or statistical worse than the result of BIBSDE in terms of the Wilcoxon rank sum test at a 0.05 significance level, respectively. Meanwhile, "≈" represents that there is no significant difference

Function	Average(standard deviation)							
	IMODE	AGSK	APGSK_IMODE	NL-SHADE-RSP	MLS-L-SHADE	EA4eig	AMCDE	BIBSDE
F1	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	(0.00E+00)≈	(0.00E+00)≈	(0.00E+00)≈	(0.00E+00)≈	(0.00E+00)≈	(0.00E+00)≈	(0.00E+00)≈	(0.00E+00)
F2	0.00E+00	0.00E+00	0.00E+00	0.00E+00	1.51E-01	1.20E+00	0.00E+00	0.00E+00
	(0.00E+00)≈	(0.00E+00)≈	(0.00E+00)≈	(0.00E+00)≈	(7.19E+00)≈	(1.86E+00)−	(0.00E+00)≈	(0.00E+00)
F3	0.00E+00	3.79E-14	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	(0.00E+00)≈	(5.45E-14)−	(0.00E+00)≈	(0.00E+00)≈	(0.00E+00)≈	(0.00E+00)≈	(0.00E+00)≈	(0.00E+00)
F4	6.70E+00	4.25E+00	3.48E+00	9.06E+00	1.86E+00	3.32E-01	2.12E+00	9.95E-02
	(1.54E+00)−	(1.38E+00)−	(1.19E+00)−	(5.05E+00)−	(7.59E+00)−	(6.58E-01)≈	(1.04E+00)−	(3.04E-01)
F5	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	(0.00E+00)≈	(0.00E+00)≈	(0.00E+00)≈	(0.00E+00)≈	(0.00E+00)≈	(0.00E+00)≈	(0.00E+00)≈	(0.00E+00)
F6	1.22E-01	4.26E-02	6.29E-02	3.48E-02	1.05E-01	8.10E-03	4.13E-02	6.89E-03
	(9.16E-02)−	(2.63E-02)−	(6.18E-02)−	(5.61E-04)−	(6.71E+00)−	(3.65E-02)−	(6.17E-02)−	(4.12E-03)
F7	0.00E+00	7.58E-15	0.00E+00	0.00E+00	1.83E-03	0.00E+00	0.00E+00	1.13E-01
	(0.00E+00)+	(4.15E-14)+	(0.00E+00)+	(0.00E+00)+	(2.16E+00)≈	(0.00E+00)+	(0.00E+00)+	(2.16E-01)
F8	8.53E-03	4.76E-02	5.66E-03	2.23E-02	1.82E+00	2.57E-03	2.32E-03	6.90E-03
	(1.06E-02)≈	(5.74E-02)−	(6.90E-03)≈	(5.84E-04)−	(1.64E+00)−	(4.74E-03)+	(2.66E-03)+	(7.77E-03)
F9	2.22E+02	2.10E+02	2.22E+02	2.22E+02	2.29E+02	1.86E+02	2.29E+02	2.06E+02
	(4.19E+01)≈	(6.17E+01)≈	(4.19E+01)≈	(1.75E+03)≈	(6.17E-01)≈	(0.00E+00)+	(0.00E+00)−	(7.00E+01)
F10	1.70E+01	9.83E+01	9.46E+01	0.00E+00	2.22E-01	1.00E+02	1.65E+01	4.25E+00
	(3.35E+01)−	(6.14E+00)−	(1.72E+01)−	(0.00E+00)+	(6.22E+00)≈	(4.02E-02)−	(3.37E+01)−	(1.82E+01)
F11	0.00E+00	2.43E-13	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	(0.00E+00)≈	(2.31E-13)−	(0.00E+00)≈	(0.00E+00)≈	(0.00E+00)≈	(0.00E+00)≈	(0.00E+00)≈	(0.00E+00)
F12	1.59E+02	1.59E+02	1.59E+02	1.60E+02	1.59E+02	1.46E+02	1.61E+02	1.59E+02
	(7.05E-01)≈	(2.89E-14)+	(7.65E-01)≈	(1.33E+00)≈	(2.55E+00)≈	(2.45E+00)+	(1.16E+00)−	(7.66E-01)
+	1	2	1	2	0	4	2	
−	3	6	3	3	3	3	5	
≈	8	4	8	7	9	5	5	

Table 9: The results for the CEC 2022 benchmark test functions when $D = 20$ with the Wilcoxon rank sum test. "+" or "−" denotes that the current result is significantly better or statistical worse than the result of BIBSDE in terms of the Wilcoxon rank sum test at a 0.05 significance level, respectively. Meanwhile, "≈" represents that there is no significant difference

Function	Average(standard deviation)							
	IMODE	AGSK	APGSK_IMODE	NL-SHADE-RSP	MLS-L-SHADE	EA4eig	AMCDE	BIBSDE
F1	0.00E+00 (0.00E+00)≈	1.89E-15 (1.04E-14)≈	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)
F2	4.27E+01 (1.46E+01)−	1.63E+00 (8.20E+00)≈	3.86E+01 (1.64E+01)−	3.62E+01 (3.39E+02)−	1.34E-01 (7.23E+00)≈	5.32E-01 (1.38E+00)−	3.04E+01 (2.18E+01)−	0.00E+00 (0.00E+00)
F3	0.00E+00 (0.00E+00)≈	1.10E-13 (2.08E-14)−	1.89E-14 (4.31E-14)−	3.79E-15 (4.31E-28)≈	0.00E+00 (0.00E+00)≈	7.58E-15 (2.88E-14)≈	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)
F4	4.87E+01 (6.49E+00)−	3.11E+01 (6.84E+00)−	2.26E+01 (3.64E+00)≈	5.19E+01 (7.50E+01)−	5.04E+00 (8.10E+00)+	8.59E+00 (3.32E+00)+	6.20E+00 (1.49E+00)+	2.09E+01 (4.05E+00)
F5	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)
F6	2.88E-01 (7.98E-02)−	2.29E-01 (1.06E-01)≈	4.72E-01 (4.77E-01)−	4.44E-01 (1.56E-01)−	1.88E-01 (6.83E+00)≈	1.23E-01 (1.55E-01)+	3.23E-01 (9.54E-02)−	2.02E-01 (6.76E-02)
F7	1.96E+00 (7.58E-01)−	1.19E+00 (1.27E+00)−	1.91E+00 (1.12E+00)−	5.40E-02 (5.93E-02)−	1.95E+00 (1.14E+00)−	2.40E+00 (3.56E+00)−	1.72E+00 (1.22E+00)−	8.07E-09
F8	1.62E+01 (4.19E+00)≈	1.76E+01 (5.42E+00)−	1.80E+01 (5.85E+00)−	1.69E+01 (3.56E+01)≈	1.61E+01 (4.46E+00)≈	1.63E+01 (7.56E+00)−	1.73E+01 (3.82E+00)−	1.51E+01 (6.05E+00)
F9	1.81E+02 (8.67E-14)≈	1.81E+02 (8.67E-14)≈	1.81E+02 (8.67E-14)≈	1.81E+02 (7.52E-27)≈	1.81E+02 (8.53E-01)≈	1.65E+02 (0.00E+00)+	1.81E+02 (8.67E-14)≈	1.81E+02 (8.67E-14)
F10	0.00E+00 (0.00E+00)≈	1.00E+02 (1.87E-02)−	1.00E+02 (3.70E-02)−	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)≈	1.08E+02 (2.93E-01)−	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)
F11	3.00E+02 (0.00E+00)≈	2.00E+01 (7.61E+01)+	2.40E+02 (1.22E+02)+	2.90E+02 (3.00E+03)≈	0.00E+00 (0.00E+00)+	3.23E+02 (4.30E+01)−	3.00E+02 (0.00E+00)≈	3.00E+02
F12	2.34E+02 (1.98E+00)−	2.33E+02 (1.34E+00)≈	2.31E+02 (1.80E+00)+	2.38E+02 (1.22E+01)−	2.32E+02 (8.63E+00)≈	2.00E+02 (3.32E-04)+	2.30E+02 (8.31E-01)+	2.33E+02 (1.90E+00)
+	0	1	2	0	2	4	2	
−	5	5	6	5	1	5	4	
≈	7	6	4	7	9	3	6	

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Table 10: The Friedman test outcome for the CEC 2020 benchmark test functions when $D = 10$

Algorithm	Ranking	Algorithm	Ranking
IMODE	4.55	MLS-L-SHADE	5.55
AGSK	5.15	EA4eig	5.40
APGSK-IMODE	5.85	AMCDE	3.70
NL-SHADE-RSP	3.15	BIBSDE	2.65

Table 11: The Friedman test outcome for the CEC 2020 benchmark test functions when $D = 20$

Algorithm	Ranking	Algorithm	Ranking
IMODE	4.05	MLS-L-SHADE	5.10
AGSK	5.45	EA4eig	6.40
APGSK-IMODE	5.30	AMCDE	3.50
NL-SHADE-RSP	2.90	BIBSDE	3.30

Table 12: The Friedman test outcome for the CEC 2022 benchmark test functions when $D = 10$

Algorithm	Ranking	Algorithm	Ranking
IMODE	4.79	MLS-L-SHADE	5.21
AGSK	5.54	EA4eig	3.67
APGSK-IMODE	4.38	AMCDE	4.33
NL-SHADE-RSP	4.46	BIBSDE	3.63

Table 13: The Friedman test outcome for the CEC 2022 benchmark test functions when $D = 20$

Algorithm	Ranking	Algorithm	Ranking
IMODE	5.17	MLS-L-SHADE	3.17
AGSK	4.88	EA4eig	4.38
APGSK-IMODE	5.58	AMCDE	4.25
NL-SHADE-RSP	5.13	BIBSDE	3.46

The results of the CEC 2020 functions when $D = 10$ can be analysed as follows. According to Table 6, all the peers are defeated by our BIBSDE from the view of the Wilconxon rank sum test. Meanwhile, according to Table 10, from the view of the Friedman test, BIBSDE ranks first.

The results of the CEC 2020 functions when $D = 20$ can be analysed as follows. According to Table 7, our algorithm is defeated by NL-SHADE-RSP but defeats the other peers from the view of the Wilconxon rank sum test. Meanwhile, according to Table 11, from the view of the Friedman test, BIBSDE ranks second and is just behind NL-SHADE-RSP.

The results of the CEC 2022 functions when $D = 10$ can be analysed as follows. According to Table 8, from the view of the Wilconxon rank sum test, our algorithm is defeated by EA4eig but defeats the other peers. Meanwhle, according to Table 12, from the view of the Friedman test, BIBSDE ranks first.

The results of the CEC 2022 functions when $D = 20$ can be analysed as follows. According to Table 9, from the view of the Wilconxon rank sum test, our algorithm is defeated by MLS-L-SHADE but defeats the other peers. Meanwhle, according to Table 13, from the view of the Friedman test, BIBSDE ranks second and is just behind MLS-L-SHADE.

A summary of the comparisons based on the Wilconxon rank sum test is given in Table 14, while that based on the Friedman test is given in Table 15. Based on an overall consideration, our BIBSDE demonstrates the best performance.

We give convergence graph of all the eight algorithms for some functions in the CEC 2022 suite. The functions are chosen because the algorithms never obtain their global optimal. Figures 1 and 2 are for $D = 10$ and $D = 20$, respectively. In the figures, for each of function, after 11 certain number of function evaluations, the average in the 30 executions of the average fitness is plotted.

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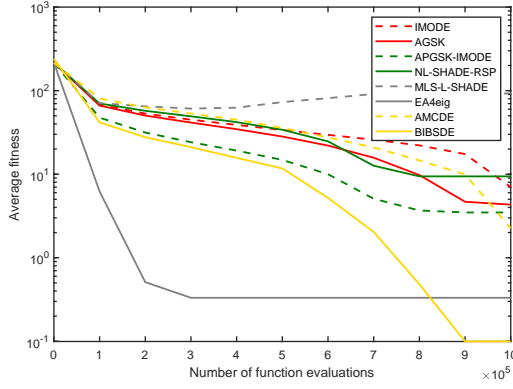
(a) F4
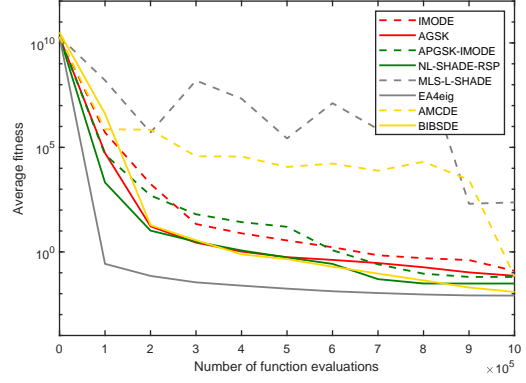
(b) F6
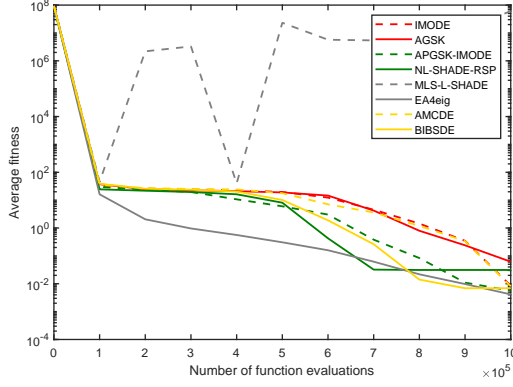
(c) F8

(d) F9

(e) F12

Figure 1: Convergence graph of the eight algorithms for the five functions in the CEC 2022 benchmark test suite when $D = 10$

(a) F4

(b) F6

(c) F7

(d) F8

(e) F9

(f) F12

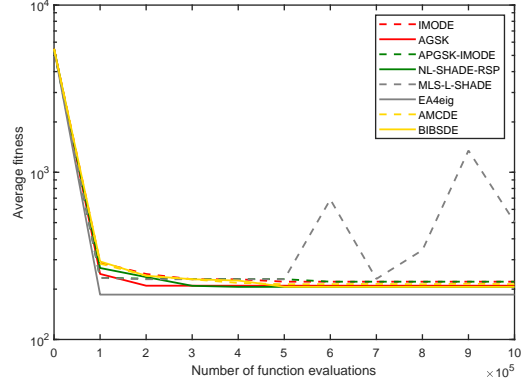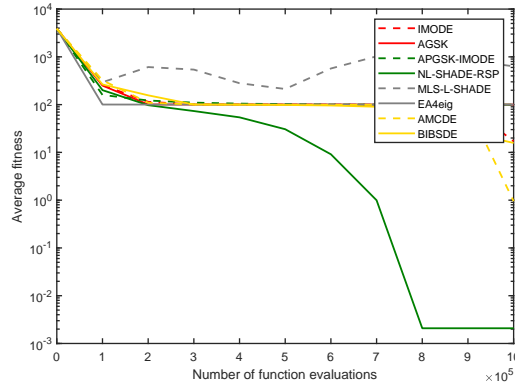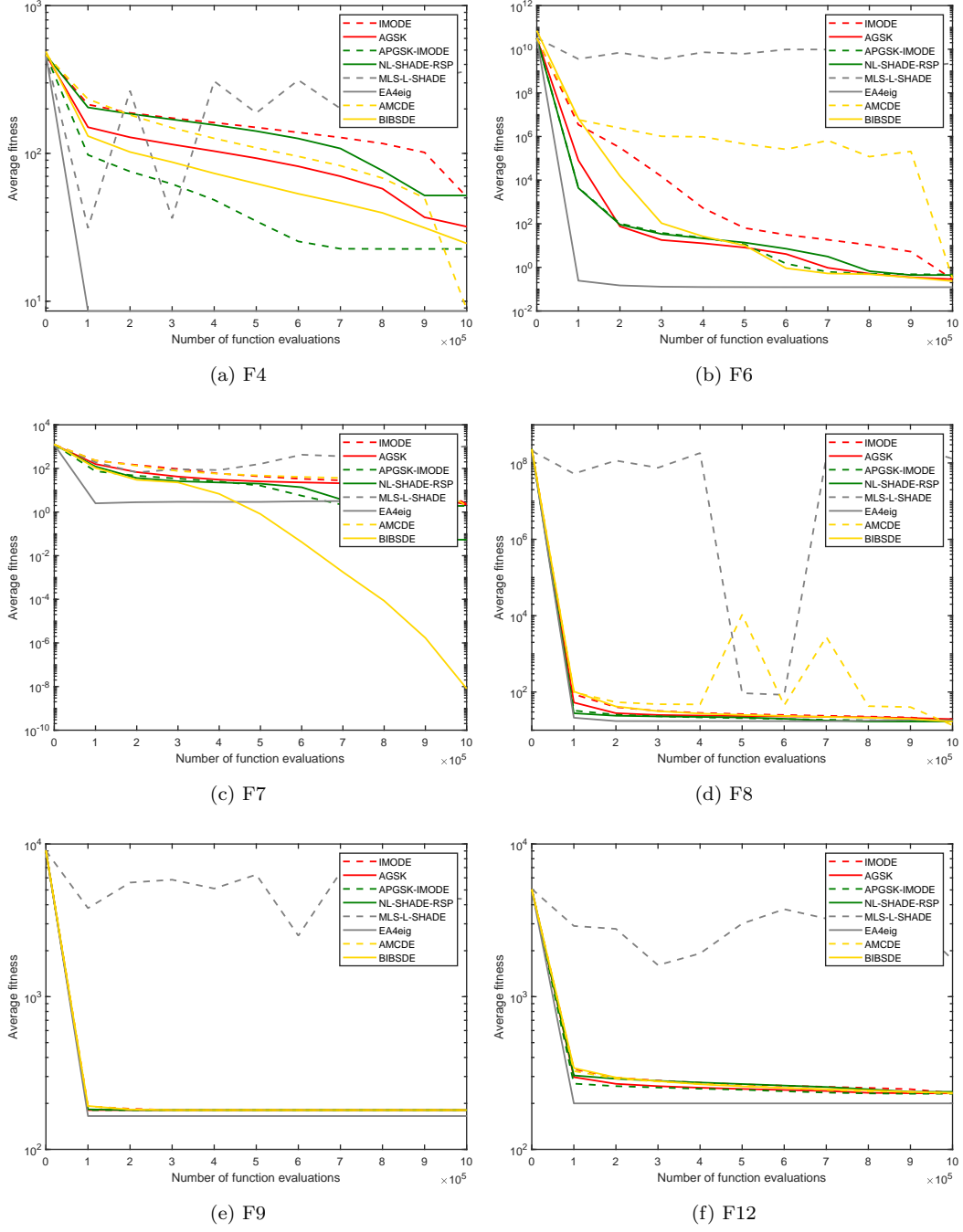Figure 2: Convergence graph of the eight algorithms for the six functions in the CEC 2022 benchmark test suite when $D = 20$

Table 14: The summary of the comparisons based on the Wilconxon rank sum test

Time(s)	IMODE	AGSK	APGSK -IMODE	NL-SHADE -RSP	MLS-L -SHADE	EA4eig	AMCDE
Defeating	4	4	4	3	3	3	4
Defeated	0	0	0	1	1	1	0

Table 15: The summary of the comparisons based on the Friedman test

Time(s)	IMODE	AGSK	APGSK -IMODE	NL-SHADE -RSP	MLS-L -SHADE	EA4eig	AMCDE
Defeating	4	4	4	3	3	4	4
Defeated	0	0	0	1	1	0	0

It can be seen that our BIBSDE shows significantly sharper convergence curve than IMODE in six cases. Meanwhile, the difference in convergence curve of the two algorithms is not significant in the rest five cases.

4.3. Comparison based on the selected CEC 2011 real-world problems

In this experiment, we select to compare our BIBSDE with IMODE, NL-SHADE-RSP, EA4eig, and AMCDE. Reasons are given below. Firstly, BIBSDE originates from IMODE. Then, NL-SHADE-RSP and EA4eig are winners of CEC competition in different years and performs better than most of the other peers in the previous experiment. Moreover, AMCDE is an algorithm published in journal. The above algorithms are compared based on the selected real-world problems with the setting in $MaxFES$. In Table 16, the results are listed with the Wilcoxon rank sum test at a 0.05 significance level.

According to Table 16, for the five problems - P1, P4, P10, P11.3, and P11.5 - out of the all eight ones, the peers and our algorithm obtain solutions much better than the reference value obtained by a recent algorithm for traditional search L-SHADE-cnEpSin-PWI [35]. That is, the problems deserve to be solved by long-term search. It can be viewed that our algorithm performs better than IMODE and NL-SHADE-RSP. Meanwhile, BIBSDE shows no advantage when compared with EA4eig. AMCDE even defeats BIBSDE narrowly. In brief, BIBSDE is still competitive compared with the peers based on the selected CEC 2011 real-world problems, although the advantage is not

Table 16: The results of the selected CEC 2011 real-world optimization problems with the Wilcoxon rank sum test. "+" or "−" denotes that the current result is significantly better or statistical worse than the result of BIBSDE in terms of the Wilcoxon rank sum test at a 0.05 significance level, respectively. Meanwhile, "≈" represents that there is no significant difference

| Function | Average(standard deviation) | | | | | |
	IMODE	NL-SHADE -RSP	EA4ig	AMCDE	BIBSDE	Reference value
P1	1.28E−13	0.00E+00	0.00E+00	0.00E+00	0.00E+00	1.83E+00
	(6.79E−13)−	(0.00E+00)≈	(0.00E+00)≈	(0.00E+00)≈	(0.00E+00)	
P3	1.15E−05	1.15E−05	1.03E−05	1.15E−05	1.15E-05	1.15E−05
	(3.74E−19)≈	(2.18E−19)≈	(3.65E−06)≈	(1.72E−21)≈	(4.42E-19)	
P4	1.41E+01	1.62E+01	1.46E+01	1.39E+01	1.46E+01	1.74E+01
	(9.85E−01)+	(3.16E+00)−	(1.78E+00)≈	(2.50E−01)+	(1.58E+00)	
P8	2.20E+02	2.20E+02	2.20E+02	2.20E+02	2.20E+02	2.20E+02
	(0.00E+00)≈	(0.00E+00)≈	(0.00E+00)≈	(0.00E+00)≈	(0.00E+00)	
P10	−2.16E+01	−2.17E+01	−2.20E+01	−2.16E+01	-2.16E+01	−2.17E+01
	(2.80E−05)≈	(9.76E−02)≈	(5.35E−01)+	(1.61E−05)≈	(3.71E-02)	
P11.3	2.67E+04	1.54E+04	1.54E+04	1.54E+04	1.54E+04	1.55E+04
	(2.17E+04)−	(1.16E+00)−	(3.53E−04)≈	(3.10E−05)≈	(1.35E-04)	
P11.4	1.82E+04	1.88E+04	1.81E+04	1.81E+04	1.81E+04	1.81E+04
	(1.16E+02)−	(1.02E+02)−	(2.18E+01)−	(3.16E+01)≈	(1.43E-01)	
P11.5	2.47E+05	3.30E+04	3.27E+04	3.27E+04	3.27E+04	3.27E+04
	(1.97E+05)−	(3.51E+01)−	(4.59E−05)≈	(1.79E+01)≈	(2.32e+00)	
+	1	0	1	1		
−	4	4	1	0		
≈	3	4	6	7		

as significant as the comparison based on the CEC benchmark test suites.

4.4. Observation on BIBSDE

To obtain BIBSDE, we improve IMODE at three aspects. In the three aspects, altogether, five revisions in algorithm behavior are made, while the value of a parameter is changed. Details are briefly reviewed below. Firstly, we replace LPSR by NLPSR. Then, for NLPSR, we propose that the reduced individuals should be those that are just better than the worst individuals in the population. Thirdly, for the mutation strategies, the ratio of best individuals - ϕ - in Equations 2-4 descends linearly during execution. Fourthly, CR is uniformly set for every position and descends linearly in the second half of execution. More importantly, we propose that, for the next generation, the current best individuals are allocated to the mutation strategy with the highest controlling ratio. In addition, we change the value of p in Equation 6 from 0.3 to 0.7.

We investigate effectiveness of the five revisions in algorithm behavior and the new value set in p by an ablation experiment. Here, the five revisions are called NLPSR (N), reduction scheme (R), ϕ descending (ϕ), CR adaptation (C), and allocation scheme (A). Meanwhile, p_{new} denotes that the new value of p is used. In this experiment, we compare results of IMODE, IMODE+R, IMODE+C, IMODE+R+C+N, IMODE+R+C+p_{new}+N, IMODE+R+C+p_{new}+N+ϕ, and our final algorithm IMODE+R+C+p_{new}+N+ϕ+A (BIBSDE) based on the CEC 2020 benchmark test functions when $D = 15$. In Table 17, results are shown. Based on Table 17, we give the Friedman test outcome in Table 18. It can be seen from Table 18 that, the more measures proposed by us are called, the better performance is obtained. Thus, our proposed measures are all beneficial.

4.5. Discussion

The first experiment based on the CEC 2020 and 2022 benchmark test suites shows that our BIBSDE converges faster than IMODE and leads to improvement on solution. Moreover, our algorithm performs better than the peers. According to the second experiment based on the selected CEC 2011 real-world problems, BIBSDE is competitive. The observation on our BIBSDE demonstrates that each measures proposed by us is effective. In detail, besides our main contribution - the scheme of best individuals allocated to the best performer among the mutation strategies, all the other measures are also meaningful.

Table 17: The results of our ablation experiment based on the CEC 2020 benchmark test functions when $D = 15$. BIBSDE-1 - BIBSDE-5 in this table denote IMODE+R, IMODE+C, IMODE+R+C+N, IMODE+R+C+p_{new}+N, and IMODE+R+C+p_{new}+N+ϕ, respectively

Function	IMODE	BIBSDE-1	BIBSDE-2	BIBSDE-3	BIBSDE-4	BIBSDE-5	BIBSDE
F1	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	(0.00E+00)	(0.00E+00)	(0.00E+00)	(0.00E+00)	(0.00E+00)	(0.00E+00)	(0.00E+00)
F2	2.57E+00	3.03E+00	4.66E+00	1.82E+00	2.20E+00	1.95E+00	4.27E-01
	(2.25E+00)	(2.65E+00)	(3.79E+00)	(1.86E+00)	(2.44E+00)	(2.13E+00)	(9.67E-01)
F3	1.61E+01	1.61E+01	1.57E+01	1.56E+01	1.56E+01	1.56E+01	1.56E+01
	(3.95E-01)	(3.94E-01)	(1.07E-01)	(1.42E-01)	(9.73E-02)	(1.07E-01)	(0.00E+00)
F4	3.49E-01	3.71E-01	4.26E-01	2.99E-01	3.70E-01	3.45E-01	3.06E-01
	(8.32E-02)	(8.19E-02)	(1.24E-01)	(8.74E-02)	(8.98E-02)	(1.04E-01)	(1.16E-01)
F5	6.79E+00	7.26E+00	5.04E+00	5.64E+00	4.97E+00	7.54E+00	5.58E+00
	(2.92E+00)	(2.83E+00)	(2.65E+00)	(2.34E+00)	(2.91E+00)	(4.09E+00)	(2.46E+00)
F6	5.66E+00	5.07E+00	1.51E+00	4.29E+00	1.69E+00	2.30E+00	8.27E-01
	(4.03E+00)	(3.66E+00)	(2.14E+00)	(4.46E+00)	(2.21E+00)	(2.81E+00)	(2.22E-01)
F7	6.57E-01	5.83E-01	4.12E-01	6.47E-01	5.97E-01	4.97E-01	6.44E-01
	(2.97E-01)	(1.91E-01)	(1.52E-01)	(2.03E-01)	(2.26E-01)	(1.55E-01)	(1.84E-01)
F8	4.18E+00	0.00E+00	3.96E+00	7.76E+00	9.55E+00	3.49E+00	2.21E+00
	(9.61E+00)	(0.00E+00)	(8.45E+00)	(1.29E+01)	(2.03E+01)	(7.43E+00)	(6.78E+00)
F9	9.33E+01	9.67E+01	9.67E+01	1.00E+02	9.09E+01	9.33E+01	1.00E+02
	(2.54E+01)	(1.83E+01)	(1.83E+01)	(2.12E-13)	(2.81E+01)	(2.54E+01)	(2.12E-13)
F10	4.00E+02	4.00E+02	4.00E+02	4.00E+02	4.00E+02	4.00E+02	4.00E+02
	(0.00E+00)	(0.00E+00)	(0.00E+00)	(0.00E+00)	(0.00E+00)	(0.00E+00)	(0.00E+00)

Table 18: The Friedman test outcome of our ablation experiment

Algorithm	Ranking
IMODE	5.00
IMODE+R	4.70
IMODE+C	4.05
IMODE+R+C+N	4.10
IMODE+R+C+p_{new}+N	3.55
IMODE+R+C+p_{new}+N+ϕ	3.50
IMODE+R+C+p_{new}+N+ϕ+A (BIBSDE)	3.10

5. Conclusion

In field of real parameter single objective optimization, long-term search has been studied for several years. Population-Based metaheuristics exhibiting low convergence velocity has been proposed for long-term search. IMODE, a DE algorithm based on the three mutation strategies, shows good performance among the population-based metaheuristics. Based on IMODE, we propose BIB-SDE. The core idea behind the proposed algorithm is regarding the best individuals as reward to the best performer among the mutation strategies. Altogether, five revisions are done by us for the proposed algorithm. Meanwhile, a parameter is set a new value. Our experiments demonstrate that BIBSDE is powerful for long-term search.

The success of BIBSDE demonstrates that the promising potential of incorporating a reward mechanism for building ensemble. However, details of reward require to be further studied. We will go on with the direction in the future.

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